

The Anderson-Moore Algorithm: Symbolic Mathematica Implementation

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September 16, 1998

Abstract

(Anderson and Moore, 1983; Anderson and Moore, 1985) describe a powerful method for solving linear saddle point models. The algorithm has proved useful in a wide array of applications including analyzing linear perfect foresight models, providing initial solutions and asymptotic constraints for nonlinear models. The algorithm solves linear problems with dozens of lags and leads and hundreds of equations in seconds. The technique works well for both symbolic algebra and numerical computation.

Although widely used at the Federal Reserve, few outside the central bank know about or have used the algorithm. This paper attempts to present the current algorithm in a more accessible format in the hope that economists outside the Federal Reserve may also find it useful. In addition, over the years there have been undocumented changes in approach that have improved the efficiency and reliability of algorithm. This paper describes the present state of development of this set of tools.

1 Problem Statement

Anderson and Moore (Anderson and Moore, 1985) outlines a procedure that computes solutions for structural models of the form

$$\sum_{i=-\tau}^{\theta} H_i x_{t+i} = 0 , \quad t \geq 0 \quad (1)$$

with initial conditions, if any, given by constraints of the form

$$x_t = x_t^{data} , \quad t = -\tau, \dots, -1 \quad (2)$$

where both τ and θ are non-negative, and x_t is an L dimensional vector with

$$\lim_{t \rightarrow \infty} x_t = 0 \quad (3)$$

The algorithm determines whether the model 1 has a unique solution, an infinity of solutions or no solutions at all.

The specification 1 is not restrictive. One can handle inhomogeneous version of equation 1 by recasting the problem in terms of deviations from a steady state value or by adding a new variable for each non-zero right hand side with an equation guaranteeing the value always equals the inhomogeneous value($x_t^{con} = x_{t-1}^{con}$ and $x_{t-1}^{con} = x^{RHS}$).

Saddle point problems combine initial conditions and asymptotic convergence to identify their solutions. The uniqueness of solutions to system 1 requires that the transition matrix characterizing the linear system have an appropriate number of explosive and stable eigenvalues(Blanchard and Kahn, 1980), and that the asymptotic linear constraints are linearly independent of explicit and implicit initial conditions(Anderson and Moore, 1985).

The solution methodology entails

1. using equation 1 to compute a state space transition matrix.
2. Computing the eigenvalues and the invariant space associated with large eigenvalues
3. Combining the constraints provided by:
 - (a) the initial conditions,
 - (b) auxiliary initial conditions identified in the computation of the transition matrix and
 - (c) the invariant space vectors

Figure 1 presents a flow chart summarizing the algorithm. For a description of a parallel implementation see (Anderson, 1997b) For a description of a continuous application see (Anderson, 1997a).

2 Algorithm

2.1 Unconstrained Auto-regression

Algorithm 1

```

1 Given  $H$ , compute the unconstrained auto-regression.
2 funct  $\mathcal{F}_1(H)$  ≡
3    $k := 0$ 
4    $\mathcal{Z}^0 := \emptyset$ 
5    $\mathcal{H}^0 := H$ 
6   while  $\mathcal{H}_\theta^k$  is singular  $\cap \text{rows}(\mathcal{Z}^k) < L(\tau + \theta)$ 
7     do
8        $U^k = \begin{bmatrix} U_Z^k \\ U_N^k \end{bmatrix} := \text{rowAnnihilator}(\mathcal{H}_\theta^k)$ 

```

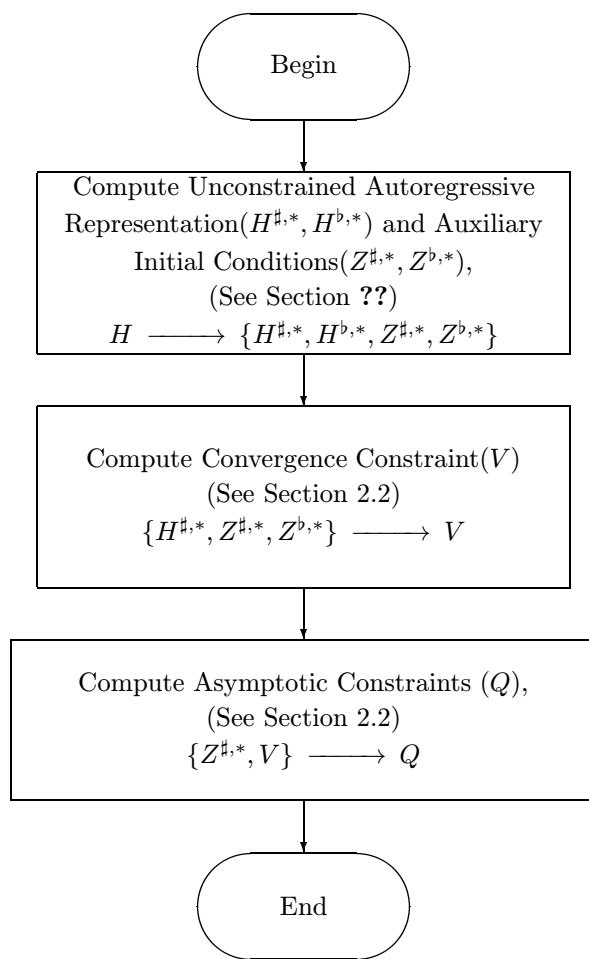


Figure 1: Algorithm Overview

```

9       $\mathcal{H}^{k+1} := \begin{bmatrix} 0 & U_Z^k \mathcal{H}_\tau^k & \dots & U_Z^k \mathcal{H}_{\theta-1}^k \\ U_N^k \mathcal{H}_\tau^k & \dots & \dots & U_N^k \mathcal{H}_\theta^k \end{bmatrix}$ 
10      $\mathcal{Z}^{k+1} := \begin{bmatrix} \mathcal{Q}^k \\ U_Z^k \mathcal{H}_\tau^k & \dots & U_Z^k \mathcal{H}_{\theta-1}^k \end{bmatrix}$ 
11      $k := k + 1$ 
12   od
13   return{ $[\mathcal{H}_{-\tau}^k \dots \mathcal{H}_\theta^k], (\Gamma \text{ or } \emptyset), \mathcal{Z}^k$ }
14 .

```

Theorem 1 Let

$$\mathcal{H} = \left[\begin{array}{ccccccccc} H_{-\tau} & & \dots & & H_\theta & & & & \\ & H_{-\tau} & & \dots & & H_\theta & & & \\ & & \ddots & & & & & & \\ & & & H_{-\tau} & & \dots & & H_\theta & \\ & & & & H_{-\tau} & & \dots & & H_\theta \\ & & & & & H_\theta & & \dots & \\ & & & & & & H_\theta & & \end{array} \right]_{\tau+\theta+1}$$

There are two cases:

- When \mathcal{H} is full rank the algorithm terminates with $Z^{\sharp*}(Z^{\flat*})$ and non-singular $H_\theta^{\sharp*}(H_\tau^{\flat*})$
- When \mathcal{H} is not full rank the algorithm terminates when some row of $[\mathcal{H}_{-\tau} \dots \mathcal{H}_\theta]$ is zero.

2.1.1 symbolicRightMostAllZeroQ

$\langle \text{symbolicRightMostAllZeroQMathematica 4} \rangle \equiv$

```

symbolicRightMostAllZeroQ[dim_,x_]:=  

With[{lilvec=Take[x,-dim]},  

If[Apply[And, (Map[Simplify[#]==0&, lilvec])],  

True,False, False]]]
◊

```

Macro referenced in scrap 14b.

2.1.2 symbolicShiftRightAndRecord

$\langle \text{symbolicShiftRightAndRecordMathematica } 5a \rangle \equiv$

```
symbolicShiftRightAndRecord[{auxiliaryConditionsSoFar_,hMatPreShifts_}]:=  
  If[Apply[Or,Map[Function[x,Apply[And,Map[#==0&,x]]],hMatPreShifts]],  
   Throw[{auxiliaryConditionsSoFar,hMatPreShifts},$zeroRow],  
   With[{dim=Length[hMatPreShifts],ldim=Length[hMatPreShifts[[1]]]},  
    FoldList[If[symbolicRightMostAllZeroQ[dim,#2],  
      {Append[#1[[1]],Drop[#2,-dim]],  
       Append[#1[[2]],RotateRight[#2,dim]]},  
      {#[[1]],Append[#1[[2]],#2]}]&,  
     {auxiliaryConditionsSoFar,{}}],hMatPreShifts][[-1]]]]
```

◇

Macro referenced in scrap 14b.

2.1.3 symbolicComputeAnnihilator

$\langle \text{symbolicComputeAnnihilatorMathematica } 5b \rangle \equiv$

```
symbolicComputeAnnihilator[amat_]:=With[{dim=Length[amat]},  
  With[{ns=RowReduce[BlockMatrix[{{amat,IdentityMatrix[dim]}}]}],  
   SubMatrix[ns,{1,dim+1},{dim, dim}]]]
```

◇

Macro referenced in scrap 14b.

2.1.4 symbolicAnnihilateRows

$\langle \text{symbolicAnnihilateRowsMathematica } 5c \rangle \equiv$

```
symbolicAnnihilateRows[hmat_]:=With[{dims=Dimensions[hmat]},  
  With[{zapper=  
    symbolicComputeAnnihilator[  
     SubMatrix[hmat,{1,dims[[2]]-dims[[1]]+1},dims[[1]][1,1]]]},  
   If[zapper=={},hmat,zapper . hmat]]]
```

◇

Macro referenced in scrap 14b.

2.1.5 symbolicAR

$\langle \text{symbolicAR}[\text{Mathematica } 6a] \rangle \equiv$

```

symbolicAR[hmat_]:= 
Catch[
FixedPoint[symbolicShiftRightAndRecord[ 
    {#[[1]],symbolicAnnihilateRows[#[[2]]]}]&,
{{},hmat},Length[hmat[[1]]], 
SameTest->(Length[#1[[1]]]==Length[#2[[1]]]&),
$zeroRow,$rankDeficiency[#1]&]
◊

```

Macro referenced in scrap 14b.

2.1.6 symbolicBiDirectionalAR

$\langle \text{symbolicBiDirectionalAR}[\text{Mathematica } 6b] \rangle \equiv$

```

symbolicBiDirectionalAR[hmat_]:= 
{symbolicAR[hmat],Map[(Map[Reverse,#])&,symbolicAR[Map[Reverse,hmat]]]}; 
◊

```

Macro referenced in scrap 14b.

$\langle \text{symbolicAIMVersion}[\text{Mathematica } 6c] \rangle \equiv$

```

symbolicAIMVersion[]:="$Revision: 1.7 $ $Date: 1998/08/31 13:07:51 $"
◊

```

Macro referenced in scrap 14b.

2.2 Invariant Space Calculations

Theorem 2 Let $\{x_t^{conv}\}$, $t = -\tau, \dots, \infty$ be a non explosive solution satisfying equation 1. Let A be the state space transition matrix for equation 1 and V be a set of invariant space vectors spanning the invariant space associated with roots of A of magnitude bigger than 1. Then for $t = 0, \dots, \infty$

$$V \begin{bmatrix} x_{t-\tau}^{conv} \\ \vdots \\ x_{t+\theta-1}^{conv} \end{bmatrix} = 0$$

Corrolary 1 Let $\{x_t\}$, $t = -\tau, \dots, \infty$ be a solution satisfying equation 1. If A has no roots with magnitude 1 then the path converges to the unique steady state if and only if

$$V \begin{bmatrix} x_{t-\tau} \\ \vdots \\ x_{t+\theta-1} \end{bmatrix} = 0$$

for some t .

Corollary 2 If A has roots with magnitude 1 then a path converges to a limit cycle(or fixed point) if and only if

$$V \begin{bmatrix} x_{t-\tau} \\ \vdots \\ x_{t+\theta-1} \end{bmatrix} = 0$$

for some t .

Algorithm 2

- 1 Given $\Gamma^{\sharp,*}, Z^{\sharp,*}, Z^{\flat,*}$,
- 2 compute vectors spanning the left invariant
- 3 space associated with large eigenvalues
- 4 funct $\mathcal{F}_2(\Gamma^{\sharp,*}, Z^{\sharp,*}, Z^{\flat,*})$
- 5 $A := \begin{bmatrix} 0 & I \\ \Gamma^{\sharp} & \end{bmatrix}$
- 6 $\{\bar{A}, \Pi, J_0\} = \text{stateSpaceReducer}(A, Z^{\sharp,*}, Z^{\flat,*})$
- 7 $\{\bar{V}, M\} := \text{leftInvariantSpaceVectors}(\bar{A})$
- 8 $V = \text{stateSpaceExpander}(\bar{V}, M, \Pi, J_0)$
- 9 .

Theorem 3 Let

$$Q = \begin{bmatrix} Z^{\sharp} \\ V \end{bmatrix} = [Q_L \quad Q_R]$$

The existence of convergent solutions depends on the magnitude of the rank of the augmented matrix

$$r_1 = \text{rank} \left(\begin{bmatrix} I & 0 & x_{\text{data}} \\ Q_L & Q_R & 0 \end{bmatrix} \right)$$

and

$$r_2 = \text{rank} \left(\begin{bmatrix} I & 0 \\ Q_L & Q_R \end{bmatrix} \right)$$

and $L(\tau + \theta)$, the number of unknowns.

1. If $r_1 > r_2$ there is no nontrivial convergent solution
2. If $r_1 = r_2 = L(\tau + \theta)$ there is a unique convergent solution
3. If $r_1 = r_2 < L(\tau + \theta)$ the system has an infinity of convergent solutions

Corrolary 3 When Q has $L\theta$ rows, Q_R is square. If Q_R is non-singular, the system has a unique solution and

$$\begin{bmatrix} B \\ B_2 \\ \vdots \\ B_\theta \end{bmatrix} = Q_R^{-1} Q_L$$

If Q_R is singular, the system has an infinity of solutions.

Corrolary 4 When Q has fewer than $L\theta$ rows, The system has an infinity of solutions.

Corrolary 5 When Q has more than $L\theta$ rows, The system has a unique nontrivial solution only for specific values of x_{data}

Algorithm 3

```

1 Given  $V, Z^{\sharp,*},$ 
2 funct  $\mathcal{F}_3(V, Z^{\sharp,*})$ 
3    $Q := \begin{bmatrix} Z^{\sharp,*} \\ V \end{bmatrix}$ 
4    $\text{cnt} = \text{noRows}(Q)$ 
5   return  $\begin{cases} \{Q, \infty\} & \text{cnt} < L\theta \\ \{Q, 0\} & \text{cnt} > L\theta \\ \{Q, \infty\} & (Q_R \text{ singular}) \\ \{-Q_R^{-1}Q, 1\} & \text{otherwise} \end{cases}$ 
6 .

```

2.3 State Space Reduction

Theorem 4 The Z_*^\sharp, Z_*^\flat span the invariant space associated with zero eigenvalue.

$$\begin{bmatrix} Z_*^\sharp \\ Z_*^\flat \end{bmatrix} A^{L(\tau+\theta)} = 0$$

Theorem 5 Suppose

$$Y \bar{A} = M Y$$

so that Y spans the invariant space associated with the eigenvalues of M , one can compute X with

$$\begin{bmatrix} X & Y \end{bmatrix}$$

spans the dominant invariant space of A . From

$$\text{vec}(X) = ((I \otimes M) - (J_0^T \otimes I))^{-1}(\Pi^T \otimes I)\text{vec}(Y)$$

Algorithm 4

- 1 Given h, H ,
- 2 asymptotic stability constraints
- 3 funct $\mathcal{F}_4(V, Z^{\sharp,*})$
- 4 .

2.3.1 symbolicEliminateInessentialLags

$$\begin{aligned} ZS^T(SAS^T) &= J_0 ZS^T \\ SAS^T &= \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} A \begin{bmatrix} S_1^T & S_2^T \end{bmatrix} = \begin{bmatrix} J_e & N \\ 0 & A_e \end{bmatrix} \\ ZS_2^T &= Q_z \begin{bmatrix} R_z \\ 0 \end{bmatrix} \\ R_{ze} &= R_z A_e \end{aligned}$$

if $k > e, \exists \alpha, \beta$ such that with

$$\begin{bmatrix} \alpha & \beta \end{bmatrix} \begin{bmatrix} R_{ze} & R_{ze} \\ 0 & R_{ze} \end{bmatrix} = 0$$

$$z_e = \beta Q_z^T ZS_2^T$$

$$\begin{aligned} Q &= I - 2VV^T \\ V^T V &= I_r \\ QX &= X - 2VV^TX \\ V &= X + E_1\Lambda \\ QX &= X - 2V(X^TX + \Lambda^T X_1) \\ X - 2[XX^TX + E_1\Lambda X^TX + X\Lambda^T X_1 + E_1\Lambda\Lambda^T X_1] \\ &\quad X - 2[XX^TX + X\Lambda^T X_1 + E_1\Lambda V^TX] \end{aligned}$$

So that will require

$$\begin{aligned} X - 2[XX^TX + X\Lambda^T X_1] &= 0 \\ I - 2X^TX - \Lambda^T X_1 &= 0 \end{aligned}$$

```

⟨symbolicEliminateInessentialLagsMathematica 10a⟩ ≡

symbolicEliminateInessentialLags[AMatrixVariableListPair_] :=
  Block[{firstzerocolumn, matrixsize},
    matrixsize = Length[AMatrixVariableListPair[[1]]];
    firstzerocolumn = FindFirstZeroColumn[
      AMatrixVariableListPair[[1]]];
    Return[
      If[IntegerQ[firstzerocolumn],
        symbolicEliminateInessentialLags[
          List[
            ((AMatrixVariableListPair[[1]])[[
              Drop[Range[1, matrixsize], {firstzerocolumn}],
              Drop[Range[1, matrixsize], {firstzerocolumn}]]]),
            Drop[AMatrixVariableListPair[[2]], {firstzerocolumn}]],
          AMatrixVariableListPair]]];
    FindFirstZeroColumn[AMatrix_] := Block[{columnsumofabs},
      columnsumofabs = Map[Apply[And, #] &,
        Map[Map[# == 0 &, #] &, Transpose[AMatrix]]];
      Return[Apply[Min, Position[columnsumofabs, True]]]]

```

◇

Macro referenced in scrap 14b.

2.3.2 symbolicSqueeze

⟨symbolicSqueezeMathematica 10b⟩ ≡

```

symbolicSqueeze[auxFAuxB_, transMat_] :=
Module[{evs, evcs, lamVal},
With[{extend = symbolicExtendToBasis[auxFAuxB]}, (*{{p1,p2},mat1,mat2}*)
With[{r12 = extend[[2]] . Transpose[extend[[1,2]]],
zeroDim = Length[extend[[1,1]]],
nonZeroDim = Length[extend[[1,2]]],
pmat = Join[extend[[1,1]], extend[[1,2]]]},
With[{ptrans = pmat . transMat. Transpose[pmat]},
{pmat, SubMatrix[ptrans, {1,1}, zeroDim*{1,1}],
SubMatrix[ptrans, {(zeroDim+1), 1}, {nonZeroDim, zeroDim}],
SubMatrix[ptrans, {(zeroDim+1){1,1}, nonZeroDim*{1,1}} - 
SubMatrix[ptrans, {(zeroDim+1), 1}, {nonZeroDim, zeroDim}] . r12 }]]]

```

◇

Macro referenced in scrap 14b.

2.3.3 symbolicMapper

$\langle \text{symbolicMapperMathematica 11a} \rangle \equiv$

```
symbolicMapper[bigPi_,bigJ0_]:=  
Function[evMat,  
    Inverse[kron[IdentityMatrix[Length[bigJ0]],evMat] -  
            kron[Transpose[bigJ0],IdentityMatrix[Length[evMat]]]] .  
            kron[Transpose[bigPi],IdentityMatrix[Length[evMat]]]]]  
◊
```

Macro referenced in scrap 14b.

2.3.4 symbolicEvExtend

$\langle \text{symbolicEvExtendMathematica 11b} \rangle \equiv$

```
symbolicEvExtend[evMat_,yMat_,mapper_,transf_]:=  
With[{xmat=mapper[evMat] . Transpose[{Flatten[Transpose[yMat]]}]},  
     BlockMatrix[{{Transpose[Partition[Flatten[xmat],  
Length[yMat](*Length[xmat]/Length[evMat]*)]],yMat}}].  
     transf]  
◊
```

Macro referenced in scrap 14b.

2.3.5 symbolicParticularLam

$\langle \text{symbolicParticularLamMathematica 11c} \rangle \equiv$

```
symbolicParticularLam[es_List,n_Integer,eMap_,transf_]:=  
{Flatten[Append[eMap[{{es[[1,n]]}}] . Transpose[{es[[2,n]]}],es[[2,n]]]]} . transf  
◊
```

Macro referenced in scrap 14b.

2.3.6 symbolicExtendToBasis

$\langle \text{symbolicTransitionMatrix 12a} \rangle \equiv$

```

symbolicTransitionMatrix[transF_]:= 
With[{nr=Length[transF]},
With[{nc=Length[transF[[1]]]-nr},
If[nc == nr,
-Inverse[SubMatrix[transF,{1,nc+1},{nr,nr}]].
SubMatrix[transF,{1,1},{nr,nc}],
BlockMatrix[{{ZeroMatrix[nc-nr,nr],IdentityMatrix[nc-nr]}, 
{-Inverse[SubMatrix[transF,{1,nc+1},{nr,nr}]]}.
SubMatrix[transF,{1,1},{nr,nc}]}}],
]]
]
◊

```

Macro referenced in scrap 14b.

$$RP^T = \begin{bmatrix} R_{11} & R_{12} \\ 0 & R_2 \end{bmatrix} = \begin{bmatrix} I & R_{12} \\ 0 & I \end{bmatrix}$$

$$(RP^T)^{-1} = \begin{bmatrix} I & -R_{12}R_2^{-1} \\ 0 & R_2^{-1} \end{bmatrix} = \begin{bmatrix} I & -R_{12} \\ 0 & I \end{bmatrix}$$

$$RP^T A(RP^T)^{-1}$$

$$J_0 = [I \ 0] RP^T A(RP^T)^{-1} \begin{bmatrix} I \\ 0 \end{bmatrix} = [I \ R_{12}] A \begin{bmatrix} -I \\ 0 \end{bmatrix} = -A_{11}$$

$$a = [0 \ I] RP^T A(RP^T)^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix} = [0 \ I] A \begin{bmatrix} -R_{12} \\ I \end{bmatrix} = A_{22} - A_{21}R_{12}$$

$$\Pi = [0 \ I] RP^T A(RP^T)^{-1} \begin{bmatrix} I \\ 0 \end{bmatrix} = [0 \ I] A \begin{bmatrix} -I \\ 0 \end{bmatrix} = -A_{21}$$

$\langle \text{symbolicExtendToBasisMathematica 12b} \rangle \equiv$

```

symbolicExtendToBasis[matRows_List]:= 
With[{spaceDim=Length[matRows[[1]]]},
With[{idim=IdentityMatrix[spaceDim]},
With[{rowEsch=Select[RowReduce[matRows],!symbolicRightMostAllZeroQ[spaceDim,#]&]},
With[{cCols=Map[Min[Flatten[Position[#,1][[1]]]]&,rowEsch]},
With[{notCCols=Complement[Range[spaceDim],cCols]},
{{idim[[cCols]], 
idim[[notCCols]],rowEsch,idim[[notCCols]]}}]]]];matRows !={}]
]
◊

```

Macro referenced in scrap 14b.

AN EXAMPLE:

Collecting the auxiliary constraints generated by the auto regression phase of

the algorithm for the example model one has:

$$\begin{bmatrix} Z_*^\sharp \\ Z_*^b \end{bmatrix} = \left[\begin{array}{cccccccccc} 0 & 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} & 1 \\ 0 & 0 & -\theta & 0 & 0 & -1 & 0 & 1 & 0 & -\gamma \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & -1 & \alpha & 1 & -\frac{1}{2} & 0 & 0 & 0 & 0 & -\frac{1}{2} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

One can extend the basis to get a non singular matrix.¹

$$\begin{bmatrix} Z \\ \bar{Z} \end{bmatrix} = \left[\begin{array}{cccccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -\frac{1}{\theta} & 0 & \frac{\gamma}{\theta} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \frac{-2\alpha}{\theta} & 2 & \frac{2\alpha\gamma-3\theta}{\theta} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right]$$

$$A = \begin{bmatrix} \rho & 2\gamma & -\gamma \\ 4\alpha & 3 & -2 \\ 2\alpha & 2 & -1 \end{bmatrix}$$

For this model, we expect one large root. Consequently, $M = [\lambda_L]$, a 1×1 matrix.

$$\Pi = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -2\gamma \\ 0 & 0 & 0 & 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{bmatrix}$$

$$J_0 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -8 + \frac{4\alpha\gamma}{\theta} - \frac{2(2\alpha\gamma-3\theta)}{\theta} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

¹The top rows are just the row-echelon form of the $Z^{\sharp,*}, Z^{b,*}$ vectors.

$$((I \otimes M) - (J_0^T \otimes I))^{-1}(\Pi^T \otimes I) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{-2\gamma}{\lambda_L} & \frac{-4}{\lambda_L} & \frac{-2}{\lambda_L} \end{bmatrix}$$

2.3.7 kron

$\langle \text{kronMathematica 14a} \rangle \equiv$

```
kron[a_,b_]:=BlockMatrix[Outer[Times,a,b]]
```

◇

Macro referenced in scrap 14b.

2.3.8 symbolicLinearAim

$\langle \text{symbolicLinearAim 14b} \rangle \equiv$

```
BeginPackage["symbolicLinearAim`", {"LinearAlgebra`MatrixManipulation`"}]
⟨symbolicRightMostAllZeroQMathematica 4⟩
⟨symbolicShiftRightAndRecordMathematica 5a⟩
⟨symbolicAnnihilateRowsMathematica 5c⟩
⟨symbolicComputeAnnihilatorMathematica 5b⟩
⟨symbolicBiDirectionalARMathematica 6b⟩
⟨symbolicARMathematica 6a⟩
⟨symbolicEliminateInessentialLagsMathematica 10a⟩
⟨symbolicSqueezeMathematica 10b⟩
⟨symbolicMapperMathematica 11a⟩
⟨symbolicEvExtendMathematica 11b⟩
⟨symbolicParticularLamMathematica 11c⟩
⟨symbolicExtendToBasisMathematica 12b⟩
⟨symbolicTransitionMatrix 12a⟩
⟨kronMathematica 14a⟩
⟨symbolicAIMVersionMathematica 6c⟩
EndPackage[]

```

◇

Macro referenced in scrap 14c.

"symbolicLinearAim.m" 14c \equiv

```
⟨symbolicLinearAim 14b⟩

```

◇

3 Test Suite

"symbolicLinearAimTest.m" 15a ≡

```
If[symbolicAR[{{0,0,0}}]==$rankDeficiency[{{}, {0, 0, 0}}],  
"passed zero row detection#00 test",  
"failed zero row detection#00 test"]
```

◇

File defined by scraps 15abcd, 16ab, 17, 18, 19.

"symbolicLinearAimTest.m" 15b ≡

```
If[symbolicAR[{{a1,b1,c1,d1,e1,f1},{a1,b1,c1,d1,e1,f1}}]  
===  
$rankDeficiency[{{}, {{a1/e1, b1/e1, c1/e1, d1/e1, 1, f1/e1},  
{0, 0, 0, 0, 0, 0}}],  
"passed zero row detection#01 test",  
"failed zero row detection#01 test"]
```

◇

File defined by scraps 15abcd, 16ab, 17, 18, 19.

"symbolicLinearAimTest.m" 15c ≡

```
If[symbolicAR[{{a1,b1,0,0,0,0},{0,0,0,a1,b1}}]  
===  
$rankDeficiency[{{a1, b1, 0, 0}, {0, 0, a1, b1}},  
{0, 0, 0, 1, b1/a1}, {0, 0, 0, 0, 0}],  
"passed zero row detection#02 test",  
"failed zero row detection#02 test"]
```

◇

File defined by scraps 15abcd, 16ab, 17, 18, 19.

"symbolicLinearAimTest.m" 15d ≡

```
If[symbolicAR[{{a,b,c}}]=={{}, {{a/c, b/c, 1}}},  
"passed AR computation#00 test",  
"failed AR computation#00 test"]
```

◇

File defined by scraps 15abcd, 16ab, 17, 18, 19.

```

"symbolicLinearAimTest.m" 16a ≡

If [symbolicAR[{{a1,b1,c1,d1,e1,f1},{a2,b2,c2,d2,e2,f2}}]
===
{{}, {{(a2*f1)/(e2*f1 - e1*f2) + (a1*f2)/(-(e2*f1) + e1*f2),
(b2*f1)/(e2*f1 - e1*f2) + (b1*f2)/(-(e2*f1) + e1*f2),
(c2*f1)/(e2*f1 - e1*f2) + (c1*f2)/(-(e2*f1) + e1*f2),
(d2*f1)/(e2*f1 - e1*f2) + (d1*f2)/(-(e2*f1) + e1*f2),
(e2*f1)/(e2*f1 - e1*f2) + (e1*f2)/(-(e2*f1) + e1*f2),
(f1*f2)/(e2*f1 - e1*f2) + (f1*f2)/(-(e2*f1) + e1*f2)},
{(a2*e1)/(-(e2*f1) + e1*f2) - (a1*e2)/(-(e2*f1) + e1*f2),
(b2*e1)/(-(e2*f1) + e1*f2) - (b1*e2)/(-(e2*f1) + e1*f2),
(c2*e1)/(-(e2*f1) + e1*f2) - (c1*e2)/(-(e2*f1) + e1*f2),
(d2*e1)/(-(e2*f1) + e1*f2) - (d1*e2)/(-(e2*f1) + e1*f2), 0,
-((e2*f1)/(-(e2*f1) + e1*f2)) + (e1*f2)/(-(e2*f1) + e1*f2)}},
"passed AR computation#01 test",
"failed AR computation#01 test"]

```

◇

File defined by scraps 15abcd, 16ab, 17, 18, 19.

```
"symbolicLinearAimTest.m" 16b ≡
```

```

If [symbolicAR[{{a1,b1,c1,d1,e1,f1},{a2,b2,c2,d2,0,0}}]
===
{{{a2, b2, c2, d2}}, {{-((a1*d2)/(-(d2*e1) + c2*f1)),
-((b1*d2)/(-(d2*e1) + c2*f1)),
-((c1*d2)/(-(d2*e1) + c2*f1)) + (a2*f1)/(-(d2*e1) + c2*f1),
-((d1*d2)/(-(d2*e1) + c2*f1)) + (b2*f1)/(-(d2*e1) + c2*f1),
-((d2*e1)/(-(d2*e1) + c2*f1)) + (c2*f1)/(-(d2*e1) + c2*f1), 0},
{(a1*c2)/(-(d2*e1) + c2*f1), (b1*c2)/(-(d2*e1) + c2*f1),
(a2*e1)/(d2*e1 - c2*f1) + (c1*c2)/(-(d2*e1) + c2*f1),
(b2*e1)/(d2*e1 - c2*f1) + (c2*d1)/(-(d2*e1) + c2*f1),
(c2*e1)/(d2*e1 - c2*f1) + (c2*e1)/(-(d2*e1) + c2*f1),
(d2*e1)/(d2*e1 - c2*f1) + (c2*f1)/(-(d2*e1) + c2*f1)}},
"passed AR computation#02 test",
"failed AR computation#02 test"]

```

◇

File defined by scraps 15abcd, 16ab, 17, 18, 19.

```

"symbolicLinearAimTest.m" 17 ≡

If[symbolicAR[{{a1,b1,c1,d1,e1,f1},{a2,b2,c2,d2,2*e1,2*f1}}]
===
{{{a1 - a2/2, b1 - b2/2, c1 - c2/2, d1 - d2/2}},

{{{(a2*(-2*d1*e1 + d2*e1))/(2*e1*(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1)),(b2*(-2*d1*e1 + d2*e1))/(2*e1*(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1)),(c2*(-2*d1*e1 + d2*e1))/(2*e1*(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1)) + (2*(a1 - a2/2)*f1)/(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1), (d2*(-2*d1*e1 + d2*e1))/(2*e1*(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1)) + (2*(b1 - b2/2)*f1)/(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1) + (-2*d1*e1 + d2*e1)/(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1) + (2*(c1 - c2/2)*f1)/(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1), (2*(d1 - d2/2)*f1)/(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1) + ((-2*d1*e1 + d2*e1)*f1)/(e1*(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1))}, {{(a2*(2*c1 - c2))/(2*(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1)),(b2*(2*c1 - c2))/(2*(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1)),((2*c1 - c2)*c2)/(2*(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1)) + (2*(a1 - a2/2)*e1)/(2*d1*e1 - d2*e1 - 2*c1*f1 + c2*f1), (2*c1 - c2)*d2)/(2*(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1)) + (2*(b1 - b2/2)*e1)/(2*d1*e1 - d2*e1 - 2*c1*f1 + c2*f1), ((2*c1 - c2)*e1)/(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1) + (2*(c1 - c2/2)*e1)/(2*d1*e1 - d2*e1 - 2*c1*f1 + c2*f1), ((2*c1 - c2)*f1)/(-2*d1*e1 + d2*e1 + 2*c1*f1 - c2*f1) + (2*(d1 - d2/2)*e1)/(2*d1*e1 - d2*e1 - 2*c1*f1 + c2*f1)}}, "passed AR computation#03 test",
"failed AR computation#03 test"]

```

◇

File defined by scraps 15abcd, 16ab, 17, 18, 19.

```

"symbolicLinearAimTest.m" 18 ≡

If[
symbolicBiDirectionalAR[{{a1,b1,c1,d1,0,0},{0,0,c2,d2,e2,f2}}]===
{{{{a1, b1, c1, d1}}, {{0, 0,
-((c2*d1)/(-(d1*e2) + c1*f2)) + (a1*f2)/(-(d1*e2) + c1*f2),
-((d1*d2)/(-(d1*e2) + c1*f2)) + (b1*f2)/(-(d1*e2) + c1*f2),
-((d1*e2)/(-(d1*e2) + c1*f2)) + (c1*f2)/(-(d1*e2) + c1*f2), 0},
{0, 0, (a1*e2)/(d1*e2 - c1*f2) + (c1*c2)/(-(d1*e2) + c1*f2),
(b1*e2)/(d1*e2 - c1*f2) + (c1*d2)/(-(d1*e2) + c1*f2),
(c1*e2)/(d1*e2 - c1*f2) + (c1*e2)/(-(d1*e2) + c1*f2),
(d1*e2)/(d1*e2 - c1*f2) + (c1*f2)/(-(d1*e2) + c1*f2)}},
{{{c2, d2, e2, f2}}, {{0, -(b1*c2)/(-(b1*c2) + a1*d2)) +
(a1*d2)/(-(b1*c2) + a1*d2),
-((c1*c2)/(-(b1*c2) + a1*d2)) + (a1*e2)/(-(b1*c2) + a1*d2),
-((c2*d1)/(-(b1*c2) + a1*d2)) + (a1*f2)/(-(b1*c2) + a1*d2), 0, 0},
{(b1*c2)/(b1*c2 - a1*d2) + (a1*d2)/(-(b1*c2) + a1*d2),
(b1*d2)/(b1*c2 - a1*d2) + (b1*d2)/(-(b1*c2) + a1*d2),
(c1*d2)/(-(b1*c2) + a1*d2) + (b1*e2)/(b1*c2 - a1*d2),
(d1*d2)/(-(b1*c2) + a1*d2) + (b1*f2)/(b1*c2 - a1*d2), 0, 0}}},
"passed AR computation#04 test",
"failed AR computation#04 test"]

```

◇

File defined by scraps 15abcd, 16ab, 17, 18, 19.

```

"symbolicLinearAimTest.m" 19 ≡

Module[{x,y,z},
With[{symBi=symbolicBiDirectionalAR[{{a1,b1,c1,d1,0,0},{0,0,c2,d2,e2,f2}}]},
With[{res=symbolicSqueeze[Join[symBi[[1,1]],symBi[[2,1]]],symbolicTransitionMatrix[symBi[[1,2]]]]},
shouldBe=
{{{0, 0, 1, 0}, {0, 0, 0, 1},
{0, 0, (c2*d1)/(-(d1*e2) + c1*f2) - (a1*f2)/(-(d1*e2) + c1*f2),
(d1*d2)/(-(d1*e2) + c1*f2) - (b1*f2)/(-(d1*e2) + c1*f2)},
{0, 0, -(a1*e2)/(d1*e2 - c1*f2) - (c1*c2)/(-(d1*e2) + c1*f2),
-(b1*e2)/(d1*e2 - c1*f2) - (c1*d2)/(-(d1*e2) + c1*f2)},
Function[{lamVal$10, evcs9, evcs10}, {{0, 0, evcs9, evcs10}}],
{{{c2*d1)/(-(d1*e2) + c1*f2) - (a1*f2)/(-(d1*e2) + c1*f2),
(d1*d2)/(-(d1*e2) + c1*f2) - (b1*f2)/(-(d1*e2) + c1*f2)},
{-(a1*e2)/(d1*e2 - c1*f2) - (c1*c2)/(-(d1*e2) + c1*f2),
-(b1*e2)/(d1*e2 - c1*f2) - (c1*d2)/(-(d1*e2) + c1*f2)}},
If[And[res[[2]][a,b,c]==shouldBe[[2]][a,b,c],Max[Abs[Flatten[res[[{1,3}]]]-shouldBe[[{1,3}]]]]]==
"passed squeeze computation#01 test",
"failed squeeze computation#01 test"]]]]

trialSubsN={alpha->2,gamma->1/10,theta->-1/5};

simsH= {{0, 0, 0, 0, 0, 0, -1, alpha, 1, -1/2, 0, 0, 0, 0, -1/2},
{0, 0, 0, -1/2, 0, 0, 0, 0, -1/2, 1, 0, 0, 0, 0, 0},
{0, 0, -theta, 0, 0, -1, 0, 1, 0, -gamma, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0}

{{af,hf},{ab,hb}}=symbolicBiDirectionalAR[simsH/.trialSubsN]
transMat=symbolicTransitionMatrix[hf];
{pm,j0,pi,lilTransMat}=symbolicSqueeze[Join[af,ab],transMat];
Drop[Eigenvalues[transMat],7]-Eigenvalues[lilTransMat]
bles=Eigensystem[Transpose[transMat]];
ules=Eigensystem[Transpose[lilTransMat]];
mapper=symbolicMapper[pi,j0];
ubigEv=symbolicParticularLam[ules,1,mapper,pm];
ubigEvs=symbolicEvExtend[DiagonalMatrix[ules[[1]]],ules[[2]],mapper,pm];

{eltrans,unZapped}=
symbolicEliminateInessentialLags[{transMat,Range[Length[transMat]]}]
zapped=Complement[Range[Length[transMat]],unZapped]
afab=Join[af,ab]
newafab=afab[[Range[Length[afab]],Join[zapped,unZapped]]];
Select[newafab,Max[Abs[#[[Range[6]]]]]== 0 &]

elafab=RowReduce[afab[[Range[Length[afab]],unZapped]]]

redafab=RowReduce[newafab][[-Range[Length[afab]-Length[zapped]],Range[-Length[unZapped],-1,1]]
{pm,j0,pi,lilTransMat}=symbolicSqueeze[redafab,eltrans];
ules=Eigensystem[Transpose[lilTransMat]];
mapper=symbolicMapper[pi,j0];
ubigEv=symbolicParticularLam[ules,1,mapper,pm];
ubigEvs=symbolicEvExtend[DiagonalMatrix[ules[[1]]],ules[[2]],mapper,pm];

symbolicHouseholderMat[x_]:=
With[{len=Length[x],sig=Sqrt[Apply[Plus,x^2]]},
With[{v={x+sig * Join[{1},Table[0,{len-1}]]}},
IdentityMatrix[len] - 2 Transpose[v]. v/((v.Transpose[v])[[1,1]])]
]]

```

4 Files

"symbolicLinearAim.m" Defined by scrap 14c.
"symbolicLinearAimTest.m" Defined by scraps 15abcd, 16ab, 17, 18, 19.

5 Macros

$\langle \text{kronMathematica} \rangle$ Referenced in scrap 14b.
 $\langle \text{symbolicAIMVersionMathematica} \rangle$ Referenced in scrap 14b.
 $\langle \text{symbolicARMathematica} \rangle$ Referenced in scrap 14b.
 $\langle \text{symbolicAnnihilateRowsMathematica} \rangle$ Referenced in scrap 14b.
 $\langle \text{symbolicBiDirectionalARMathematica} \rangle$ Referenced in scrap 14b.
 $\langle \text{symbolicComputeAnnihilatorMathematica} \rangle$ Referenced in scrap 14b.
 $\langle \text{symbolicEliminateInessentialLagsMathematica} \rangle$ Referenced in scrap 14b.
 $\langle \text{symbolicEvExtendMathematica} \rangle$ Referenced in scrap 14b.
 $\langle \text{symbolicExtendToBasisMathematica} \rangle$ Referenced in scrap 14b.
 $\langle \text{symbolicLinearAim} \rangle$ Referenced in scrap 14c.
 $\langle \text{symbolicMapperMathematica} \rangle$ Referenced in scrap 14b.
 $\langle \text{symbolicParticularLamMathematica} \rangle$ Referenced in scrap 14b.
 $\langle \text{symbolicRightMostAllZeroQMathematica} \rangle$ Referenced in scrap 14b.
 $\langle \text{symbolicShiftRightAndRecordMathematica} \rangle$ Referenced in scrap 14b.
 $\langle \text{symbolicSqueezeMathematica} \rangle$ Referenced in scrap 14b.
 $\langle \text{symbolicTransitionMatrix} \rangle$ Referenced in scrap 14b.

6 Identifiers

kronMathematica: 14a, 14b.
symbolicAnnihilateRowsMathematica: 5c, 14b.
symbolicARMathematica: 6a, 14b.
symbolicBiDirectionalARMathematica: 6b, 14b.
symbolicComputeAnnihilatorMathematica: 5b, 14b.
symbolicEvExtendMathematica: 11b, 14b.
symbolicExtendToBasisMathematica: 12b, 14b.
symbolicLinearAim: 14b, 14c.
symbolicMapperMathematica: 11a, 14b.
symbolicParticularLamMathematica: 11c, 14b.
symbolicRightMostAllZeroQMathematica: 4, 14b.
symbolicShiftRightAndRecordMathematica: 5a, 14b.
symbolicSqueezeMathematica: 10b, 14b.
symbolicTransitionMatrix: 12a, 14b, 19.

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